Math 115

Winter 2017

Lecture 15

Chapter 6: Working with Rational Expressions

Rational Expression: $\dfrac{\text{Polynomial}}{\text{Polynomial}}$

$$\frac{2x-8}{3x-12} \quad , \quad \frac{x^2-2x-15}{x^2-25} \quad , \quad \frac{2x^2+5x-7}{3x^2-4x-8}$$

To Simplify a rational expression:

1) factor numerator completely.

2) factor denominator

3) Cross-out any common factors.

Simplify

$$\frac{2x-8}{3x-12} = \frac{2(x-4)}{3(x-4)} = \boxed{\frac{2}{3}}$$

Simplify: $\frac{4x-16}{x^2-4} = \frac{4(x-4)}{(x+2)(x-2)}$

No Common factors
irreducible.

$$\frac{x^2-2x-15}{x^2-25}$$

$$= \frac{(x-5)(x+3)}{(x+5)(x-5)} = \boxed{\frac{x+3}{x+5}}$$

Simplify

$$\frac{3x^2-5x+2}{4x^2+3x-7} = \frac{(3x-2)(x-1)}{(x-1)(4x+7)} = \frac{3x-2}{4x+7}$$

$$\frac{x^2-9}{x^3-27} \xrightarrow{A^2-B^2} = \frac{(x+3)(x-3)}{(x-3)(x^2+3x+9)} = \boxed{\frac{x+3}{x^2+3x+9}}$$

$$\searrow A^3-B^3$$

Evaluate: $\dfrac{x+5}{x-2}$ for $x=-5$, $x=2$

for $x=-5$

$\dfrac{-5+5}{-5-2} = \dfrac{0}{-7} = 0$

for $x=2$

$\dfrac{2+5}{2-2} = \dfrac{7}{0}$

Undefined

Evaluate $\dfrac{4x}{x^2-9}$ for $x=3$, $x=-3$

$\dfrac{4(3)}{3^2-9} = \dfrac{12}{0}$ undefined

$\dfrac{4(-3)}{(-3)^2-9} = \dfrac{-12}{0}$ undefined.

Any value that makes the denominator zero is called excluded value.

To find excluded Values:

1) Denominator $=0$

2) Solve

Find excluded Values

$\dfrac{x+7}{x^2-8x+15}$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x=3 \qquad x=5$

E.V. : 3 & 5

Find excluded Values:

$$\frac{x-2}{x^2-25}$$

$x^2 - 25 = 0$

$(x-5)(x+5)=0$

by Z.P.R.

$x-5=0$, $x+5=0$

$x=5$ $x=-5$

E.V. ⇒ ±5

$$\frac{2x+3}{x^2-x-20}$$

$x^2-x-20=0$

$(x+4)(x-5)=0$

by Z.P.R.

$x+4=0$ $x-5=0$

$x=-4$ $x=5$

E.V. ⇒ -4 & 5

$$\frac{3x-5}{2x^2-3x-5}$$

$2x^2-3x-5=0$

$(2x-5)(x+1)=0$

by Z.P.R.

$2x-5=0$, $x+1=0$

$x=5/2$ $x=-1$

E.V. ⇒ $5/2$, -1

Final exam Sample question:

① Simplify: $\dfrac{x^2-5x}{2x^2-50} = \dfrac{x(x-5)}{2(x+5)(x-5)}$

$$= \boxed{\frac{x}{2(x+5)}}$$

② find all excluded Values: $\dfrac{x^2-8x+3}{x^2-20x+100}$

$x^2-20x+100=0$

$(x-10)(x-10)=0$

by Z.P.R.

$x-10=0$

$\boxed{x=10}$

E.V. ⇒ 10

January 30, 2017

How to multiply rational expressions:

1) factor numerators comp.

2) factor denominators comp.

3) Cross out any common factors.

$$\frac{2x-10}{x^2-25} \cdot \frac{x^2+7x+10}{x^2+2x} = \frac{2(x-5)}{(x-5)(x+5)} \cdot \frac{(x+5)(x+2)}{x(x+2)}$$

$$= \boxed{\frac{2}{x}}$$

Multiply

$$\frac{x^2+8x+15}{x^2-9} \cdot \frac{x^2-4x+3}{x^2-25} = \frac{(x+5)(x+3)}{(x+3)(x-3)} \cdot \frac{(x-3)(x-1)}{(x-5)(x+5)}$$

$$= \frac{x-1}{x-5}$$

Divide:

$$\frac{4x-12}{7x+10} \div \frac{x^2-9}{49x^2-100} = \frac{4x-12}{7x+10} \cdot \frac{49x^2-100}{x^2-9}$$

$(7x)^2 - (10)^2$

$$= \frac{4(x-3)}{7x+10} \cdot \frac{(7x+10)(7x-10)}{(x+3)(x-3)} = \boxed{\frac{4(7x-10)}{x+3}}$$

① find all excluded values: $\dfrac{x^2+3x+2}{x^2+x-30}$

$x^2+x-30=0$

$(x+6)(x-5)=0$ by Z.P.R.   E.V. $\Rightarrow -6$ & 5

$x=-6, \ x=5$

② Simplify: $\dfrac{6y^2-36y+54}{4y^2-36} = \dfrac{6(y^2-6y+9)}{4(y^2-9)} = \dfrac{\overset{3}{\cancel{6}}(y-3)\cancel{(y-3)}}{\underset{2}{\cancel{4}}(y-3)(y+3)}$

$= \boxed{\dfrac{3(y-3)}{2(y+3)}}$

③ Multiply: $\dfrac{6y-12}{2y^2+3y-2} \cdot \dfrac{y^2-4}{8y-8}$

$= \dfrac{\overset{3}{\cancel{6}}(y-2)}{(2y-1)\cancel{(y+2)}} \cdot \dfrac{(y-2)\cancel{(y+2)}}{\underset{4}{\cancel{8}}(y-1)} = \boxed{\dfrac{3(y-2)^2}{4(y-1)(2y-1)}}$

④ Divide: $\dfrac{4x^4}{x^2-1} \div \dfrac{2x^3}{x^2-2x+1}$

$= \dfrac{4x^4}{x^2-1} \cdot \dfrac{x^2-2x+1}{2x^3}$

$= \dfrac{\overset{2}{\cancel{4}}\overset{x}{\cancel{x^4}}}{(x+1)\cancel{(x-1)}} \cdot \dfrac{\cancel{(x-1)}(x-1)}{\cancel{2x^3}}$

$= \dfrac{2x(x-1)}{x+1}$

Add & subtract rational expressions
with same denominators

1) Repeat the deno.

2) Add or subtract numerators

3) Simplify, Simplify, Simplify

Simplify:

$$\dfrac{x^2}{x-5} - \dfrac{25}{x-5}$$

$$= \dfrac{x^2 - 25}{x-5}$$

$$= \dfrac{(x+5)(x-5)}{x-5}$$

$$= \boxed{x+5}$$

Simplify

$$\dfrac{x^2+3x}{x^2-4} - \dfrac{x}{x^2-4} = \dfrac{x^2+3x-x}{x^2-4}$$

$$\dfrac{x^2+3x}{x^2-25} - \dfrac{-2x}{x^2-25}$$

$$= \dfrac{x^2+2x}{x^2-4} = \dfrac{x(x+2)}{(x+2)(x-2)}$$

$$= \dfrac{x^2+3x+2x}{x^2-25} = \dfrac{x^2+5x}{x^2-25}$$

$$= \boxed{\dfrac{x}{x-2}}$$

$$= \dfrac{x(x+5)}{(x-5)(x+5)} = \boxed{\dfrac{x}{x-5}}$$

$$\frac{x+9}{x-5} + \frac{6-4x}{x-5}$$

$$= \frac{x+9+6-4x}{x-5}$$

$$= \frac{-3x+15}{x-5}$$

$$= \frac{-3(x-5)}{x-5} = \boxed{-3}$$

$$\frac{6x-3}{x^2-x-12} - \frac{2x-15}{x^2-x-12}$$

$$= \frac{6x-3-2x+15}{x^2-x-12}$$

$$= \frac{4x+12}{x^2-x-12}$$

$$= \frac{4(x+3)}{(x+3)(x-4)} = \boxed{\frac{4}{x-4}}$$

Simplify

$$\frac{3x-2}{2x} + \frac{2(x+1)}{2x}$$

$$= \frac{3x-2+2(x+1)}{2x} = \frac{3x-2+2x+2}{2x}$$

$$= \frac{5x}{2x} = \boxed{\frac{5}{2}}$$

$$\frac{2x}{x^2-16} - \frac{2}{x-4}$$

$$= \frac{2x}{(x+4)(x-4)} - \frac{2\cdot(x+4)}{(x-4)(x+4)} = \frac{2x-2(x+4)}{(x-4)(x+4)}$$

$$= \frac{2x-2x-8}{(x-4)(x+4)} = \boxed{\frac{-8}{(x-4)(x+4)}}$$

$$\frac{4(x+4)}{(x+3)(x-2)(x+4)} + \frac{2(x-2)}{(x+3)(x+4)(x-2)}$$

$$= \frac{4(x+4) + 2(x-2)}{(x+3)(x-2)(x+4)} = \frac{4x+16+2x-4}{(x+3)(x-2)(x+4)}$$

$$= \boxed{\frac{6x+12}{(x+3)(x-2)(x+4)}} = \boxed{\frac{6(x+2)}{(x+3)(x-2)(x+4)}}$$

Quadratic formula

If $ax^2 + bx + c = 0$, $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\rightarrow x = \frac{-2+8}{2} \quad , \quad x = \frac{2-8}{2}$$

$$x = \frac{6}{2} \quad x = \frac{-10}{2}$$

$$\boxed{x=3} \quad \boxed{x=5}$$

Solve $x^2 + 2x = 15$

$$x^2 + 2x - 15 = 0 \qquad a=1, \quad b=2, \quad c=-15$$

$$b^2 - 4ac = (2)^2 - 4(1)(-15) = 4 + 60 = \boxed{64}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{64}}{2(1)} = \frac{-2 \pm 8}{2} = \qquad \{5,3\}$$

Solve by Q-formula:

$2x^2 = x + 3$

$a = 2$ $b = -1$ $c = -3$

$2x^2 - x - 3 = 0$

$b^2 - 4ac = (-1)^2 - 4(2)(-3)$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= 1 + 24 = 25$

$x = \dfrac{-(-1) \pm \sqrt{25}}{2(2)} = \dfrac{1 \pm 5}{4}$

$x = \dfrac{1+5}{4}$ $x = \dfrac{1-5}{4}$

$x = \dfrac{6}{4}$ $x = \dfrac{-4}{4}$

$\left\{ \dfrac{3}{2}, -1 \right\}$

$\boxed{x = \dfrac{3}{2}}$ $\boxed{x = -1}$

Solve by Q-formula

$4x^2 + 9 = 12x$

$b^2 - 4ac = (-12)^2 - 4(4)(9)$

$4x^2 + 9 - 12x = 0$

$= 144 - 144$

$4x^2 - 12x + 9 = 0$

$= 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 4$ $b = -12$ $c = 9$

$x = \dfrac{-(-12) \pm \sqrt{0}}{2(4)}$

$\left\{ \dfrac{3}{2} \right\}$

$x = \dfrac{12 \pm 0}{8} = \dfrac{12}{8} = \boxed{\dfrac{3}{2}}$

find x:

x   a   c   x+4

x

b

x+2

$b^2 - 4ac = (-4)^2 - 4(1)(-12)$

$= 16 + 48$

$= 64$
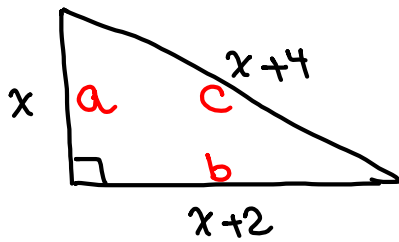
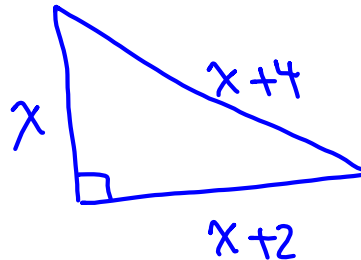$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{4 \pm \sqrt{64}}{2}$

Right Triangle

$a^2 + b^2 = c^2$

$x^2 + (x+2)^2 = (x+4)^2$

$x^2 + (x+2)(x+2) = (x+4)(x+4)$

$x^2 + x^2 + 2x + 2x + 4 = x^2 + 4x + 4x + 16$

$2x^2 + 4x + 4 = x^2 + 8x + 16$

$2x^2 + 4x + 4 - x^2 - 8x - 16 = 0$

$x^2 - 4x - 12 = 0$

$a = 1, \quad b = -4, \quad c = -12$

$x = \dfrac{4 \pm 8}{2}$

$x = \dfrac{4+8}{2}$ $x = \dfrac{4-8}{2}$

$\boxed{x = 6}$ $x = 2$

x   x+4

x+2

$\longrightarrow \{6\}$

x a c $2x+3$

b

$2x+2$
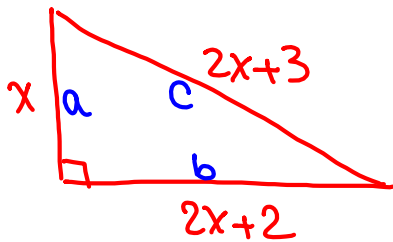
Right Triangle
Use Pythagorean thrm

$a^2 + b^2 = c^2$

$x^2 + (2x+2)^2 = (2x+3)^2$

$x^2 + (2x+2)(2x+2) = (2x+3)(2x+3)$

find x

$b^2 - 4ac = (-4)^2 - 4(1)(-5)$ $x^2 + 4x^2 + 4x + 4x + 4 = 4x^2 + 6x + 6x + 9$

$= 36$ $x^2 + 8x + 4 - 12x - 9 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x^2 - 4x - 5 = 0$

$x = \dfrac{4 \pm \sqrt{36}}{2}$ $a = 1, \ b = -4, \ c = -5$

$x = \dfrac{4 \pm 6}{2}$

$x = \dfrac{4+6}{2}, \quad x = \dfrac{4-6}{2}$

$x = \dfrac{10}{2} \qquad x = \dfrac{-2}{2}$

$x = 5 \qquad \cancel{x = -1}$

$\hookrightarrow \{5\}$

x $2x+3$

$2x+2$

5 13

12
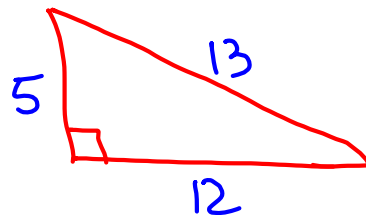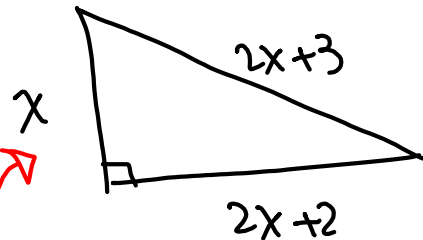
$5^2 + 12^2 = 13^2$

$25 + 144 = 169 \checkmark$

SG 15 & 16

Due Tuesday